

Learning Objectives

By using the information and exercises in this chapter you will be able to:

1. Understand the different units with which engineers work.
2. Understand the relationship between charge and current and how to use both in a variety of applications.
3. Understand voltage and how it can be used in a variety of applications.
4. Develop an understanding of power and energy and their relationship with current and voltage.
5. Begin to understand the volt-amp characteristics of a variety of circuit elements.
6. Begin to understand an organized approach to problem solving and how it can be used to assist in your efforts to solve circuit problems.

1.1 Introduction

Electric circuit theory and electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are built. Many branches of electrical engineering, such as power, electric machines, control, electronics, communications, and instrumentation, are based on electric circuit theory. Therefore, the basic electric circuit theory course is the most important course for an electrical engineering student, and always an excellent starting point for a beginning student in electrical engineering education. Circuit theory is also valuable to students specializing in other branches of the physical sciences because circuits are a good model for the study of energy systems in general, and because of the applied mathematics, physics, and topology involved.

In electrical engineering, we are often interested in communicating or transferring energy from one point to another. To do this requires an interconnection of electrical devices. Such interconnection is referred to as an *electric circuit*, and each component of the circuit is known as an *element*.

An electric circuit is an interconnection of electrical elements.

A simple electric circuit is shown in Fig. 1.1. It consists of three basic elements: a battery, a lamp, and connecting wires. Such a simple circuit can exist by itself; it has several applications, such as a flashlight, a search light, and so forth.

A complicated real circuit is displayed in Fig. 1.2, representing the schematic diagram for a radio receiver. Although it seems complicated, this circuit can be analyzed using the techniques we cover in this book. Our goal in this text is to learn various analytical techniques and computer software applications for describing the behavior of a circuit like this.

Electric circuits are used in numerous electrical systems to accomplish different tasks. Our objective in this book is not the study of various uses and applications of circuits. Rather, our major concern is the analysis of the circuits. By the analysis of a circuit, we mean a study of the behavior

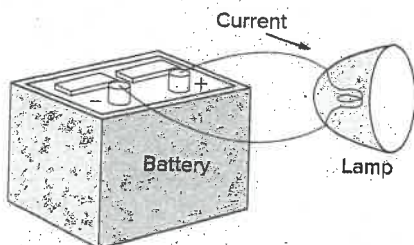


Figure 1.1
A simple electric circuit.

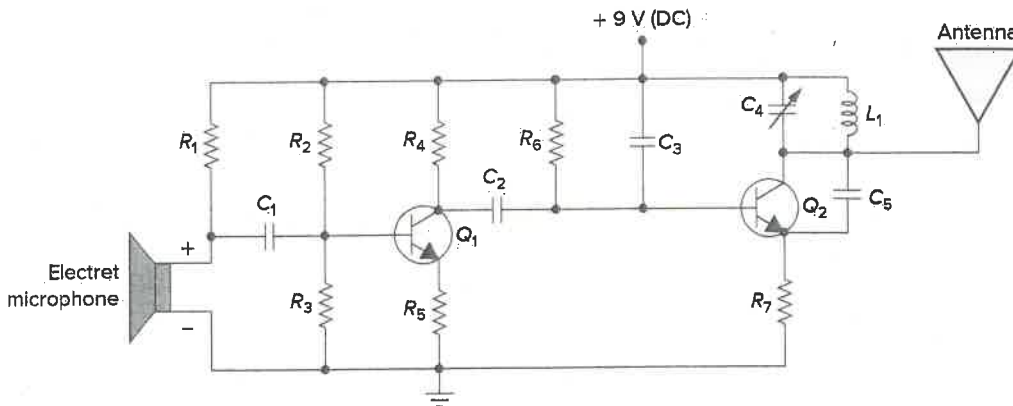


Figure 1.2
Electric circuit of a radio transmitter.

of the circuit: How does it respond to a given input? How do the interconnected elements and devices in the circuit interact?

We commence our study by defining some basic concepts. These concepts include charge, current, voltage, circuit elements, power, and energy. Before defining these concepts, we must first establish a system of units that we will use throughout the text.

1.2 Systems of Units

As electrical engineers, we must deal with measurable quantities. Our measurements, however, must be communicated in a standard language that virtually all professionals can understand, irrespective of the country in which the measurement is conducted. Such an international measurement language is the International System of Units (SI), adopted by the General Conference on Weights and Measures in 1960. In this system, there are seven base units from which the units of all other physical quantities can be derived. Table 1.1 shows six base units and one derived unit (the coulomb) that are related to this text. SI units are commonly used in electrical engineering.

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. Table 1.2 shows the SI prefixes and their symbols. For example, the following are expressions of the same distance in meters (m):

$$600,000,000 \text{ mm} \quad 600,000 \text{ m} \quad 600 \text{ km}$$

TABLE 1.1

Six basic SI units and one derived unit relevant to this text.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Charge	coulomb	C

TABLE 1.2

The SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

1.3 Charge and Current

The concept of electric charge is the underlying principle for explaining all electrical phenomena. Also, the most basic quantity in an electric circuit is the *electric charge*. We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge e on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

The following points should be noted about electric charge:

1. The coulomb is a large unit for charges. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. Thus realistic or laboratory values of charges are on the order of pC, nC, or μC .¹
2. According to experimental observations, the only charges that occur in nature are integral multiples of the electronic charge $e = -1.602 \times 10^{-19}$ C.
3. The *law of conservation of charge* states that charge can neither be created nor destroyed, only transferred. Thus, the algebraic sum of the electric charges in a system does not change.

We now consider the flow of electric charges. A unique feature of electric charge or electricity is the fact that it is mobile; that is, it can be transferred from one place to another, where it can be converted to another form of energy.

When a conducting wire (consisting of several atoms) is connected to a battery (a source of electromotive force), the charges are compelled to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates electric current. It is conventional to take the current flow as the movement of positive charges. That is, opposite to the flow of negative charges, as Fig. 1.3 illustrates. This convention was introduced by Benjamin Franklin (1706–1790), the American scientist and inventor. Although we now know that current in metallic conductors is due to negatively charged electrons, we will follow the universally accepted convention that current is the net flow of positive charges. Thus,

Electric current is the time rate of change of charge, measured in amperes (A).

Mathematically, the relationship between current i , charge q , and time t is

$$i \triangleq \frac{dq}{dt} \quad (1.1)$$

¹ However, a large power supply capacitor can store up to 0.5 C of charge.

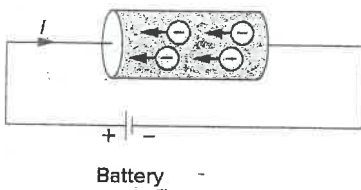


Figure 1.3
Electric current due to flow of electronic charge in a conductor.

A convention is a standard way of describing something so that others in the profession can understand what we mean. We will be using IEEE conventions throughout this book.

Historical

Andre-Marie Ampere (1775–1836), a French mathematician and physicist, laid the foundation of electrodynamics. He defined the electric current and developed a way to measure it in the 1820s.

Born in Lyons, France, Ampere at age 12 mastered Latin in a few weeks, as he was intensely interested in mathematics and many of the best mathematical works were in Latin. He was a brilliant scientist and a prolific writer. He formulated the laws of electromagnetics. He invented the electromagnet and the ammeter. The unit of electric current, the ampere, was named after him.



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where current is measured in amperes (A), and

$$1 \text{ ampere} = 1 \text{ coulomb/second}$$

The charge transferred between time t_0 and t is obtained by integrating both sides of Eq. (1.1). We obtain

$$Q \triangleq \int_{t_0}^t i \, dt \quad (1.2)$$

The way we define current as i in Eq. (1.1) suggests that current need not be a constant-valued function. As many of the examples and problems in this chapter and subsequent chapters suggest, there can be several types of current; that is, charge can vary with time in several ways.

There are different ways of looking at direct current and alternating current. The best definition is that there are two ways that current can flow: It can always flow in the same direction, where it does not reverse direction, in which case we have *direct current* (dc). These currents can be constant or time varying. If the current flows in both directions, then we have *alternating current* (ac).

A direct current (dc) flows only in one direction and can be constant or time varying.

By convention, we will use the symbol I to represent a constant current. If the current varies with respect to time (either dc or ac) we will use the symbol i . A common use of this would be the output of a rectifier (dc) such as $i(t) = |5 \sin(377t)|$ amps or a sinusoidal current (ac) such as $i(t) = 160 \sin(377t)$ amps.

An alternating current (ac) is a current that changes direction with respect to time.

An example of alternating current (ac) is the current you use in your house to run the air conditioner, refrigerator, washing machine, and other electric appliances. Figure 1.4 depicts two common examples of dc

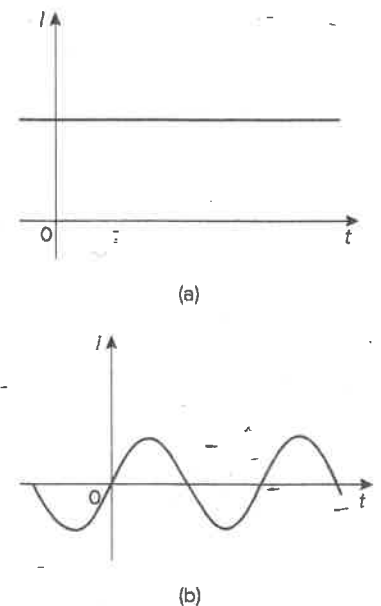


Figure 1.4
Two common types of current: (a) direct current (dc), (b) alternating current (ac).

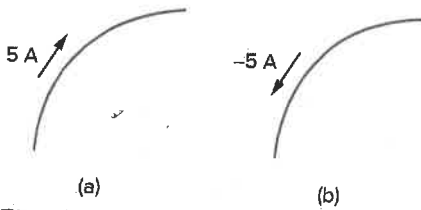


Figure 1.5
Conventional current flow: (a) positive current flow, (b) negative current flow.

(coming from a battery) and ac (coming from your home outlets). We will consider other types later in the book.

Once we define current as the movement of charge, we expect current to have an associated direction of flow. As mentioned earlier, the direction of current flow is conventionally taken as the direction of positive charge movement. Based on this convention, a current of 5 A may be represented positively or negatively as shown in Fig. 1.5. In other words, a negative current of -5 A flowing in one direction as shown in Fig. 1.5(b) is the same as a current of $+5$ A flowing in the opposite direction.

Example 1.1

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have -1.602×10^{-19} C/electron \times 4,600 electrons = -7.369×10^{-16} C

Practice Problem 1.1

Calculate the amount of charge represented by 10 billion protons.

Answer: 1.6021×10^{-9} C.

Example 1.2

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt} (5t \sin 4\pi t) \text{ mC/s} = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$,

$$i = 5 \sin 2\pi + 10\pi \cos 2\pi = 0 + 10\pi = 31.42 \text{ mA}$$

Practice Problem 1.2

If in Example 1.2, $q = (20 - 15t - 10e^{-3t})$ mC, find the current at $t = 1.0$ s.

Answer: -13.506 mA.

Example 1.3

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$\begin{aligned} Q &= \int_{t=1}^2 i \, dt = \int_1^2 (3t^2 - t) \, dt \\ &= \left(t^3 - \frac{t^2}{2} \right) \Big|_1^2 = (8 - 2) - \left(1 - \frac{1}{2} \right) = 5.5 \text{ C} \end{aligned}$$

The current flowing through an element is

$$i = \begin{cases} 8 \text{ A}, & 0 < t < 1 \\ 8t^2 \text{ A}, & t > 1 \end{cases}$$

Calculate the charge entering the element from $t = 0$ to $t = 2$ s.

Answer: 26.67 C.

Practice Problem 1.3

1.4 Voltage

As explained briefly in the previous section, to move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as *voltage* or *potential difference*. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from b to a ; mathematically,

$$v_{ab} \triangleq \frac{dw}{dq} \quad (1.3)$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage v_{ab} or simply v is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745–1827), who invented the first voltaic battery. From Eq. (1.3), it is evident that

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

Thus,

Voltage (or potential difference) is the energy required to move a unit charge from a reference point (–) to another point (+), measured in volts (V).

Figure 1.6 shows the voltage across an element (represented by a rectangular block) connected to points a and b . The plus (+) and minus (–) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) Point a is at a potential of v_{ab} volts higher than point b , or (2) the potential at point a with respect to point b is v_{ab} . It follows logically that in general

$$v_{ab} = -v_{ba} \quad (1.4)$$

For example, in Fig. 1.7, we have two representations of the same voltage. In Fig. 1.7(a), point a is +9 V above point b ; in Fig. 1.7(b), point b is –9 V above point a . We may say that in Fig. 1.7(a), there is a 9-V *voltage drop* from a to b or equivalently a 9-V *voltage rise* from b to a . In other words, a voltage drop from a to b is equivalent to a voltage rise from b to a .

Current and voltage are the two basic variables in electric circuits. The common term *signal* is used for an electric quantity such as a current or a voltage (or even electromagnetic wave) when it is used for

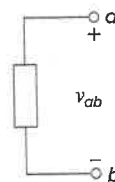


Figure 1.6
Polarity of voltage v_{ab} .

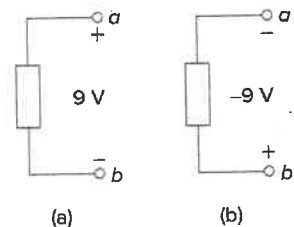


Figure 1.7
Two equivalent representations of the same voltage v_{ab} : (a) Point a is 9 V above point b ; (b) point b is –9 V above point a .

Historical



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Alessandro Antonio Volta (1745–1827), an Italian physicist, invented the electric battery—which provided the first continuous flow of electricity—and the capacitor.

Born into a noble family in Como, Italy, Volta was performing electrical experiments at age 18. His invention of the battery in 1796 revolutionized the use of electricity. The publication of his work in 1800 marked the beginning of electric circuit theory. Volta received many honors during his lifetime. The unit of voltage or potential difference, the volt, was named in his honor.

Keep in mind that electric current is always *through* an element and that electric voltage is always *across* the element or between two points.

conveying information. Engineers prefer to call such variables signals rather than mathematical functions of time because of their importance in communications and other disciplines. Like electric current, a constant voltage is called a *dc voltage* and is represented by V , whereas a sinusoidally time-varying voltage is called an *ac voltage* and is represented by v . A dc voltage is commonly produced by a battery; ac voltage is produced by an electric generator.

1.5 Power and Energy

Although current and voltage are the two basic variables in an electric circuit, they are not sufficient by themselves. For practical purposes, we need to know how much *power* an electric device can handle. We all know from experience that a 100-watt bulb gives more light than a 60-watt bulb. We also know that when we pay our bills to the electric utility companies, we are paying for the electric *energy* consumed over a certain period of time. Thus, power and energy calculations are important in circuit analysis.

To relate power and energy to voltage and current, we recall from physics that:

Power is the time rate of expending or absorbing energy, measured in watts (W).

We write this relationship as

$$p \triangleq \frac{dw}{dt} \quad (1.5)$$

where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s). From Eqs. (1.1), (1.3), and (1.5), it follows that

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \tag{1.6}$$

or

$$p = vi \tag{1.7}$$

The power p in Eq. (1.7) is a time-varying quantity and is called the *instantaneous power*. Thus, the power absorbed or supplied by an element is the product of the voltage across the element and the current through it. If the power has a + sign, power is being delivered to or absorbed by the element. If, on the other hand, the power has a - sign, power is being supplied by the element. But how do we know when the power has a negative or a positive sign?

Current direction and voltage polarity play a major role in determining the sign of power. It is therefore important that we pay attention to the relationship between current i and voltage v in Fig. 1.8(a). The voltage polarity and current direction must conform with those shown in Fig. 1.8(a) in order for the power to have a positive sign. This is known as the *passive sign convention*. By the passive sign convention, current enters through the positive polarity of the voltage. In this case, $p = +vi$ or $vi > 0$ implies that the element is absorbing power. However, if $p = -vi$ or $vi < 0$, as in Fig. 1.8(b), the element is releasing or supplying power.

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Unless otherwise stated, we will follow the passive sign convention throughout this text. For example, the element in both circuits of Fig. 1.9 has an absorbing power of +12 W because a positive current enters the positive terminal in both cases. In Fig. 1.10, however, the element is supplying power of +12 W because a positive current enters the negative terminal. Of course, an absorbing power of -12 W is equivalent to a supplying power of +12 W. In general,

$$+\text{Power absorbed} = -\text{Power supplied}$$

In fact, the *law of conservation of energy* must be obeyed in any electric circuit. For this reason, the algebraic sum of power in a circuit, at any instant of time, must be zero:

$$\sum p = 0 \tag{1.8}$$

This again confirms the fact that the total power supplied to the circuit must balance the total power absorbed.

From Eq. (1.6), the energy absorbed or supplied by an element from time t_0 to time t is

$$w = \int_{t_0}^t p \, dt = \int_{t_0}^t vi \, dt \tag{1.9}$$

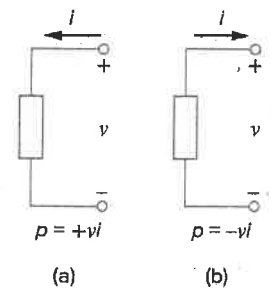


Figure 1.8 Reference polarities for power using the passive sign convention: (a) absorbing power, (b) supplying power.

When the voltage and current directions conform to Fig. 1.8(b), we have *active sign convention* and $p = +vi$.

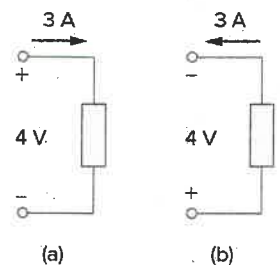


Figure 1.9 Two cases of an element with an absorbing power of 12 W: (a) $p = 4 \times 3 = 12$ (b) $p = 4 \times 3 = 12$ W.

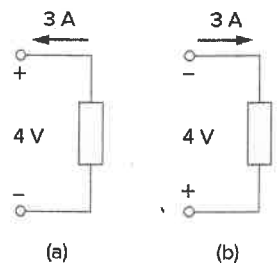


Figure 1.10 Two cases of an element with a supplying power of 12 W: (a) $p = -4 \times 3 = -12$ (b) $p = -4 \times 3 = -12$ W.

Energy is the capacity to do work, measured in joules (J).

The electric power utility companies measure energy in watt-hours (Wh), where

$$1 \text{ Wh} = 3,600 \text{ J}$$

Example 1.4

An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

The total charge is

$$\Delta q = i \Delta t = 2 \times 10 = 20 \text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115 \text{ V}$$

Practice Problem 1.4

To move charge q from point b to point a requires 100 J. Find the voltage drop v_{ab} (the voltage at a positive with respect to b) if: (a) $q = 5 \text{ C}$, (b) $q = -10 \text{ C}$.

Answer: (a) 20 V, (b) -10 V.

Example 1.5

Find the power delivered to an element at $t = 3 \text{ ms}$ if the current entering its positive terminal is

$$i = 5 \cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$, (b) $v = 3 di/dt$.

Solution:

(a) The voltage is $v = 3i = 15 \cos 60\pi t$; hence, the power is

$$p = vi = 75 \cos^2 60\pi t \text{ W}$$

At $t = 3 \text{ ms}$,

$$p = 75 \cos^2 (60\pi \times 3 \times 10^{-3}) = 75 \cos^2 0.18\pi = 53.48 \text{ W}$$

(b) We find the voltage and the power as

$$v = 3 \frac{di}{dt} = 3(-60\pi)5 \sin 60\pi t = -900\pi \sin 60\pi t \text{ V}$$

$$p = vi = -4500\pi \sin 60\pi t \cos 60\pi t \text{ W}$$

At $t = 3 \text{ ms}$,

$$p = -4500\pi \sin 0.18\pi \cos 0.18\pi \text{ W}$$

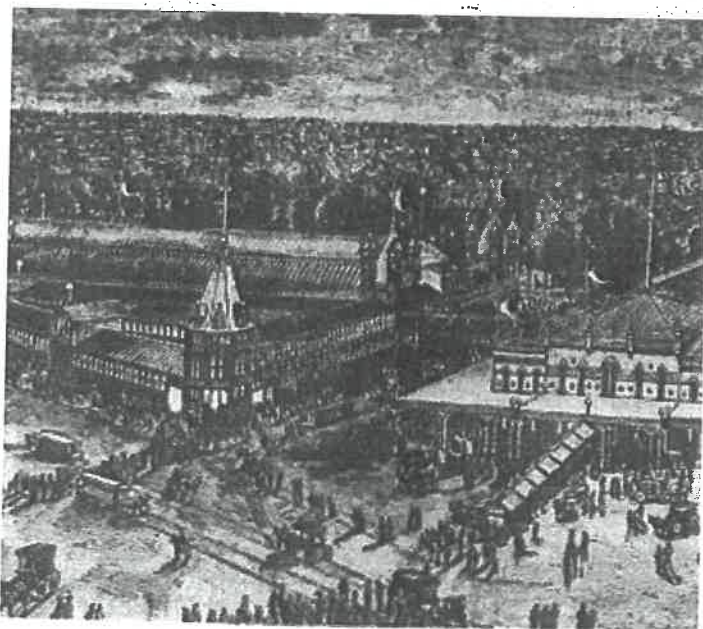
$$= -14137.167 \sin 32.4^\circ \cos 32.4^\circ = -6.396 \text{ kW}$$

Historical

1884 Exhibition In the United States, nothing promoted the future of electricity like the 1884 International Electrical Exhibition. Just imagine a world without electricity, a world illuminated by candles and gaslights, a world where the most common transportation was by walking and riding on horseback or by horse-drawn carriage. Into this world an exhibition was created that highlighted Thomas Edison and reflected his highly developed ability to promote his inventions and products. His exhibit featured spectacular lighting displays powered by an impressive 100-kW “Jumbo” generator.

Edward Weston’s dynamos and lamps were featured in the United States Electric Lighting Company’s display. Weston’s well known collection of scientific instruments was also shown.

Other prominent exhibitors included Frank Sprague, Elihu Thompson, and the Brush Electric Company of Cleveland. The American Institute of Electrical Engineers (AIEE) held its first technical meeting on October 7–8 at the Franklin Institute during the exhibit. AIEE merged with the Institute of Radio Engineers (IRE) in 1964 to form the Institute of Electrical and Electronics Engineers (IEEE).



Source: IEEE History Center



Find the power delivered to the element in Example 1.5 at $t = 5$ ms if the current remains the same but the voltage is: (a) $v = 6i$ V,

$$(b) v = \left(6 + 10 \int_0^t i \, dt \right) \text{ V.}$$

Answer: (a) 51.82 W, (b) 18.264 watts.

Practice Problem 1.5

Example 1.6

How much energy does a 100-W electric bulb consume in two hours?

Solution:

$$w = pt = 100 \text{ (W)} \times 2 \text{ (h)} \times 60 \text{ (min/h)} \times 60 \text{ (s/min)}$$

$$= 720,000 \text{ J} = 720 \text{ kJ}$$

This is the same as

$$w = pt = 100 \text{ W} \times 2 \text{ h} = 200 \text{ Wh}$$

Practice Problem 1.6

A home electric heater draws 12 A when connected to a 115 V outlet. How much energy is consumed by the heater over a period of 24 hours?

Answer: 33.12 k watt-hours

1.6 Circuit Elements

As we discussed in Section 1.1, an element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are two types of elements found in electric circuits: *passive* elements and *active* elements. An active element is capable of generating energy while a passive element is not. Examples of passive elements are resistors, capacitors, and inductors. Typical active elements include generators, batteries, and operational amplifiers. Our aim in this section is to gain familiarity with some important active elements.

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources: independent and dependent sources.

An **ideal independent source** is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

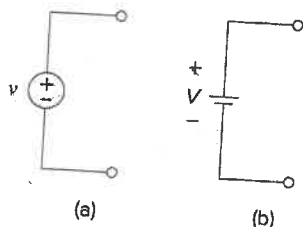


Figure 1.11
Symbols for independent voltage sources:
(a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources. Figure 1.11 shows the symbols for independent voltage sources. Notice that both symbols in Fig. 1.11(a) and (b) can be used to represent a dc voltage source, but only the symbol in Fig. 1.11(a) can be used for a time-varying voltage source. Similarly, an ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source. That is, the current source delivers to the circuit whatever voltage

is necessary to maintain the designated current. The symbol for an independent current source is displayed in Fig. 1.12, where the arrow indicates the direction of current i .

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.13. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

Dependent sources are useful in modeling elements such as transistors, operational amplifiers, and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.14, where the voltage $10i$ of the voltage source depends on the current i through element C . Students might be surprised that the value of the dependent voltage source is $10i$ V (and not $10i$ A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+ -) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

It should be noted that an ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated, whereas an ideal current source will produce the necessary voltage to ensure the stated current flow. Thus, an ideal source could in theory supply an infinite amount of energy. It should also be noted that not only do sources supply power to a circuit, they can absorb power from a circuit too. For a voltage source, we know the voltage but not the current supplied or drawn by it. By the same token, we know the current supplied by a current source but not the voltage across it.

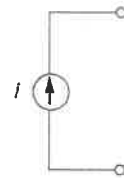


Figure 1.12
Symbol for independent current source.

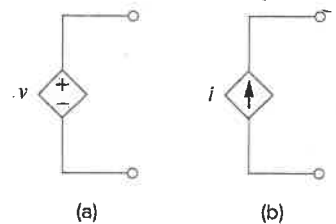


Figure 1.13
Symbols for: (a) dependent voltage source, (b) dependent current source.

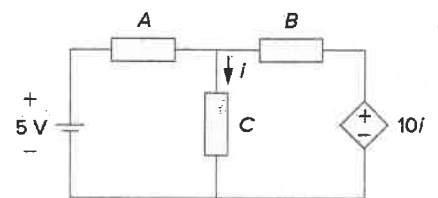


Figure 1.14
The source on the right-hand side is a current-controlled voltage source.

Calculate the power supplied or absorbed by each element in Fig. 1.15.

Solution:

We apply the sign convention for power shown in Figs. 1.8 and 1.9. For p_1 , the 5-A current is out of the positive terminal (or into the negative terminal); hence,

$$p_1 = 20(-5) = -100 \text{ W} \quad \text{Supplied power}$$

For p_2 and p_3 , the current flows into the positive terminal of the element in each case.

$$p_2 = 12(5) = 60 \text{ W} \quad \text{Absorbed power}$$

$$p_3 = 8(6) = 48 \text{ W} \quad \text{Absorbed power}$$

Example 1.7

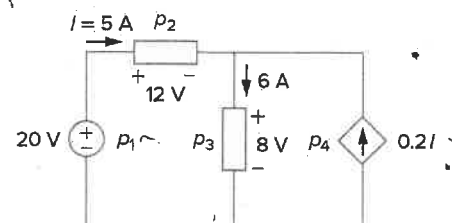


Figure 1.15
For Example 1.7.

For p_4 , we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3 since both the passive element and the dependent source are connected to the same terminals. (Remember that voltage is always measured across an element in a circuit.) Since the current flows out of the positive terminal,

$$p_4 = 8(-0.2I) = 8(-0.2 \times 5) = -8 \text{ W} \quad \text{Supplied power}$$

We should observe that the 20-V independent voltage source and $0.2I$ dependent current source are supplying power to the rest of the network, while the two passive elements are absorbing power. Also,

$$p_1 + p_2 + p_3 + p_4 = -100 + 60 + 48 - 8 = 0$$

In agreement with Eq. (1.8), the total power supplied equals the total power absorbed.

Practice Problem 1.7

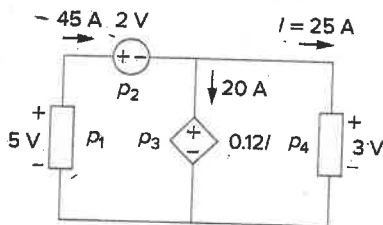


Figure 1.16
For Practice Prob. 1.7.

Compute the power absorbed or supplied by each component of the circuit in Fig. 1.16.

Answer: $p_1 = -225 \text{ W}$, $p_2 = 90 \text{ W}$, $p_3 = 60 \text{ W}$, $p_4 = 75 \text{ W}$.

1.7 † Applications²

In this section, we will consider two practical applications of the concepts developed in this chapter. The first one deals with the TV picture tube and the other with how electric utilities determine your electric bill.

1.7.1 TV Picture Tube

One important application of the motion of electrons is found in both the transmission and reception of TV signals. At the transmission end, a TV camera reduces a scene from an optical image to an electrical signal. Scanning is accomplished with a thin beam of electrons in an iconoscope camera tube.

At the receiving end, the image is reconstructed by using a cathode-ray tube (CRT) located in the TV receiver.³ The CRT is depicted in Fig. 1.17. Unlike the iconoscope tube, which produces an electron beam of constant intensity, the CRT beam varies in intensity according to the incoming signal. The electron gun, maintained at a high potential, fires the electron beam. The beam passes through two sets of plates for vertical and horizontal deflections so that the spot on the screen where the beam strikes can move right and left and up and down. When the electron beam strikes the fluorescent screen, it gives off light at that spot. Thus, the beam can be made to "paint" a picture on the TV screen.

²The dagger sign preceding a section heading indicates the section that may be skipped, explained briefly, or assigned as homework.

³Modern TV tubes use a different technology.

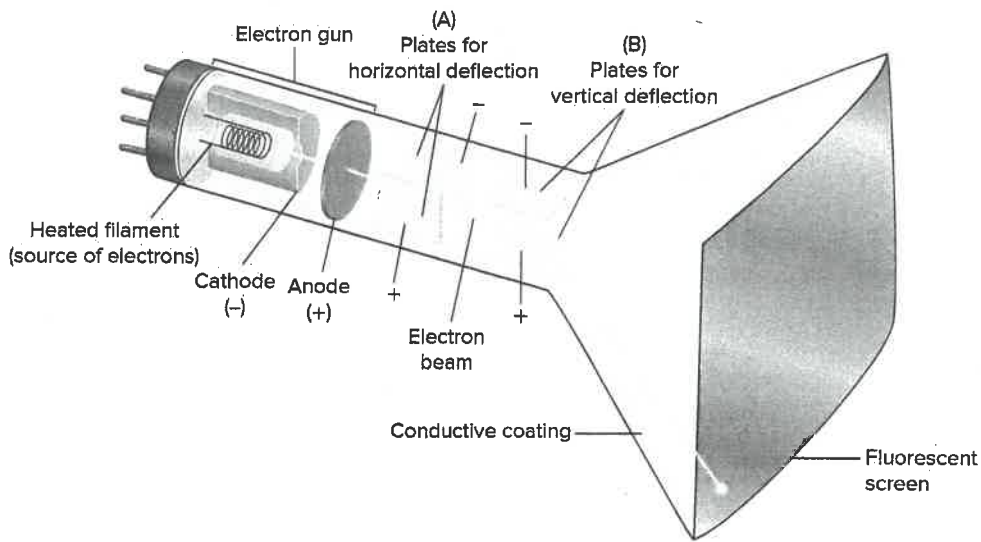


Figure 1.17
Cathode-ray tube.

Historical

Karl Ferdinand Braun and Vladimir K. Zworykin

Karl Ferdinand Braun (1850–1918), of the University of Strasbourg, invented the Braun cathode-ray tube in 1879. This then became the basis for the picture tube used for so many years for televisions. It is still the most economical device today, although the price of flat-screen systems is rapidly becoming competitive. Before the Braun tube could be used in television, it took the inventiveness of **Vladimir K. Zworykin** (1889–1982) to develop the iconoscope so that the modern television would become a reality. The iconoscope developed into the orthicon and the image orthicon, which allowed images to be captured and converted into signals that could be sent to the television receiver. Thus, the television camera was born.

The electron beam in a TV picture tube carries 10^{15} electrons per second. As a design engineer, determine the voltage V_0 needed to accelerate the electron beam to achieve 4 W.

Example 1.8

Solution:

The charge on an electron is

$$e = -1.6 \times 10^{-19} \text{ C}$$

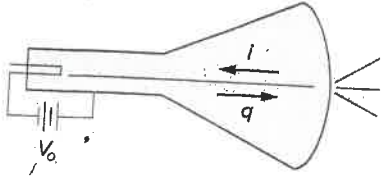


Figure 1.18

A simplified diagram of the cathode-ray tube; for Example 1.8.

If the number of electrons is n , then $q = ne$ and

$$i = \frac{dq}{dt} = e \frac{dn}{dt} = (-1.6 \times 10^{-19})(10^{15}) = -1.6 \times 10^{-4} \text{ A}$$

The negative sign indicates that the current flows in a direction opposite to electron flow as shown in Fig. 1.18, which is a simplified diagram of the CRT for the case when the vertical deflection plates carry no charge. The beam power is

$$p = V_0 i \quad \text{or} \quad V_0 = \frac{p}{i} = \frac{4}{1.6 \times 10^{-4}} = 25,000 \text{ V}$$

Thus, the required voltage is 25 kV.

Practice Problem 1.8

If an electron beam in a TV picture tube carries 10^{13} electrons/second and is passing through plates maintained at a potential difference of 25 kV, calculate the power in the beam.

Answer: 40 mW.

1.7.2 Electricity Bills

The second application deals with how an electric utility company charges their customers. The cost of electricity depends upon the amount of energy consumed in kilowatt-hours (kWh). (Other factors that affect the cost include demand and power factors; we will ignore these for now.) However, even if a consumer uses no energy at all, there is a minimum service charge the customer must pay because it costs money to stay connected to the power line. As energy consumption increases, the cost per kWh drops. It is interesting to note the average monthly consumption of household appliances for a family of five, shown in Table 1.3.

TABLE 1.3

Typical average monthly consumption of household appliances.

Appliance	kWh consumed	Appliance	kWh consumed
Water heater	500	Washing machine	120
Freezer	100	Stove	100
Lighting	100	Dryer	80
Dishwasher	35	Microwave oven	25
Electric iron	15	Personal computer	12
TV	10	Radio	8
Toaster	4	Clock	2

Example 1.9

A homeowner consumes 700 kWh in January. Determine the electricity bill for the month using the following residential rate schedule:

- Base monthly charge of \$12.00.
- First 100 kWh per month at 16 cents/kWh.
- Next 200 kWh per month at 10 cents/kWh.
- Over 300 kWh per month at 6 cents/kWh.

Solution:

We calculate the electricity bill as follows.

$$\begin{aligned}
 \text{Base monthly charge} &= \$12.00 \\
 \text{First 100 kWh @ } \$0.16/\text{kWh} &= \$16.00 \\
 \text{Next 200 kWh @ } \$0.10/\text{kWh} &= \$20.00 \\
 \text{Remaining 400 kWh @ } \$0.06/\text{kWh} &= \$24.00 \\
 \text{Total charge} &= \$72.00 \\
 \text{Average cost} &= \frac{\$72}{100 + 200 + 400} = 10.2 \text{ cents/kWh}
 \end{aligned}$$

Referring to the residential rate schedule in Example 1.9, calculate the average cost per kWh if only 260 kWh are consumed in July when the family is on vacation most of the time.

Practice Problem 1.9

Answer: 16.923 cents/kWh.

1.8 † Problem Solving

Although the problems to be solved during one's career will vary in complexity and magnitude, the basic principles to be followed remain the same. The process outlined here is the one developed by the authors over many years of problem solving with students, for the solution of engineering problems in industry, and for problem solving in research.

We will list the steps simply and then elaborate on them.

1. Carefully **define** the problem.
 2. **Present** everything you know about the problem.
 3. Establish a set of **alternative** solutions and determine the one that promises the greatest likelihood of success.
 4. **Attempt** a problem solution.
 5. **Evaluate** the solution and check for accuracy.
 6. Has the problem been solved **satisfactorily**? If so, present the solution; if not, then return to step 3 and continue through the process again.
1. *Carefully define the problem.* This may be the most important part of the process, because it becomes the foundation for all the rest of the steps. In general, the presentation of engineering problems is

Now we can solve for v_1 .

$$8 \left[\frac{v_1 - 5}{2} + \frac{v_1 - 0}{8} + \frac{v_1 + 3}{4} \right] = 0$$

$$\text{leads to } (4v_1 - 20) + (v_1) + (2v_1 + 6) = 0$$

$$7v_1 = +14, \quad v_1 = +2 \text{ V}, \quad i_{8\Omega} = \frac{v_1}{8} = \frac{2}{8} = \mathbf{0.25 \text{ A}}$$

5. *Evaluate the solution and check for accuracy.* We can now use Kirchhoff's voltage law (KVL) to check the results.

$$i_1 = \frac{v_1 - 5}{2} = \frac{2 - 5}{2} = -\frac{3}{2} = -1.5 \text{ A}$$

$$i_2 = i_{8\Omega} = 0.25 \text{ A}$$

$$i_3 = \frac{v_1 + 3}{4} = \frac{2 + 3}{4} = \frac{5}{4} = 1.25 \text{ A}$$

$$i_1 + i_2 + i_3 = -1.5 + 0.25 + 1.25 = 0 \quad (\text{Checks.})$$

Applying KVL to loop 1,

$$\begin{aligned} -5 + v_{2\Omega} + v_{8\Omega} &= -5 + (-i_1 \times 2) + (i_2 \times 8) \\ &= -5 + [-(-1.5)2] + (0.25 \times 8) \\ &= -5 + 3 + 2 = 0 \quad (\text{Checks.}) \end{aligned}$$

Applying KVL to loop 2,

$$\begin{aligned} -v_{8\Omega} + v_{4\Omega} - 3 &= -(i_2 \times 8) + (i_3 \times 4) - 3 \\ &= -(0.25 \times 8) + (1.25 \times 4) - 3 \\ &= -2 + 5 - 3 = 0 \quad (\text{Checks.}) \end{aligned}$$

So we now have a very high degree of confidence in the accuracy of our answer.

6. *Has the problem been solved satisfactorily? If so, present the solution; if not, then return to step 3 and continue through the process again.* This problem has been solved satisfactorily.

The current through the 8- Ω resistor is 0.25 A flowing down through the 8- Ω resistor.

Practice Problem 1.10

Try applying this process to some of the more difficult problems at the end of the chapter.

1.9 Summary

1. An electric circuit consists of electrical elements connected together.
2. The International System of Units (SI) is the international measurement language, which enables engineers to communicate their results. From the seven principal units, the units of other physical quantities can be derived.

3. Current is the rate of charge flow past a given point in a given direction.

$$i = \frac{dq}{dt}$$

4. Voltage is the energy required to move 1 C of charge from a reference point (-) to another point (+).

$$v_{ab} = \frac{dw}{dq}$$

5. Power is the energy supplied or absorbed per unit time. It is also the product of voltage and current.

$$p = \frac{dw}{dt} = vi$$

6. According to the passive sign convention, power assumes a positive sign when the current enters the positive polarity of the voltage across an element.
7. An ideal voltage source produces a specific potential difference across its terminals regardless of what is connected to it. An ideal current source produces a specific current through its terminals regardless of what is connected to it.
8. Voltage and current sources can be dependent or independent. A dependent source is one whose value depends on some other circuit variable.
9. Two areas of application of the concepts covered in this chapter are the TV picture tube and electricity billing procedure.

Review Questions

- 1.1 One millivolt is one millionth of a volt.
(a) True (b) False
- 1.2 The prefix *micro* stands for:
(a) 10^6 (b) 10^3 (c) 10^{-3} (d) 10^{-6}
- 1.3 The voltage 2,000,000 V can be expressed in powers of 10 as:
(a) 2 mV (b) 2 kV (c) 2 MV (d) 2 GV
- 1.4 A charge of 2 C flowing past a given point each second is a current of 2 A.
(a) True (b) False
- 1.5 The unit of current is:
(a) coulomb (b) ampere
(c) volt (d) joule
- 1.6 Voltage is measured in:
(a) watts (b) amperes
(c) volts (d) joules per second
- 1.7 A 4-A current charging a dielectric material will accumulate a charge of 24 C after 6 s.
(a) True (b) False
- 1.8 The voltage across a 1.1-kW toaster that produces a current of 10 A is:
(a) 11 kV (b) 1100 V (c) 110 V (d) 11 V
- 1.9 Which of these is not an electrical quantity?
(a) charge (b) time (c) voltage
(d) current (e) power
- 1.10 The dependent source in Fig. 1.22 is:
(a) voltage-controlled current source
(b) voltage-controlled voltage source
(c) current-controlled voltage source
(d) current-controlled current source

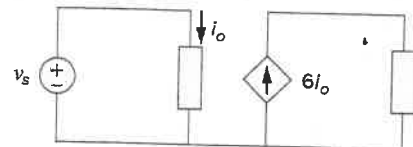


Figure 1.22
For Review Question 1.10.

Answers: 1.1b, 1.2d, 1.3c, 1.4a, 1.5b, 1.6c, 1.7a, 1.8c, 1.9b, 1.10d.

Problems

Section 1.3 Charge and Current

- 1.1 How much charge is represented by these number of electrons?
- 6.482×10^{17}
 - 1.24×10^{18}
 - 2.46×10^{19}
 - 1.628×10^{20}
- 1.2 Determine the current flowing through an element if the charge flow is given by
- $q(t) = (3t + 8) \text{ mC}$
 - $q(t) = (8t^2 + 4t - 2) \text{ C}$
 - $q(t) = (3e^{-t} - 5e^{-2t}) \text{ nC}$
 - $q(t) = 10\sin(120\pi t) \text{ pC}$
 - $q(t) = 20e^{-4t} \cos(50t) \text{ } \mu\text{C}$
- 1.3 Find the charge $q(t)$ flowing through a device if the current is:
- $i(t) = 3 \text{ A}$, $q(0) = 1 \text{ C}$
 - $i(t) = (2t + 5) \text{ mA}$, $q(0) = 0$
 - $i(t) = 20 \cos(10t + \pi/6) \text{ } \mu\text{A}$, $q(0) = 2 \text{ } \mu\text{C}$
 - $i(t) = 10e^{-30t} \sin 40t \text{ A}$, $q(0) = 0$
- 1.4 A current of 7.4 A flows through a conductor. Calculate how much charge passes through any cross-section of the conductor in 20 seconds.
- 1.5 Determine the total charge transferred over the time interval of $0 \leq t \leq 10 \text{ s}$ when $i(t) = \frac{1}{2}t \text{ A}$.
- 1.6 The charge entering a certain element is shown in Fig. 1.23. Find the current at:
- $t = 1 \text{ ms}$
 - $t = 6 \text{ ms}$
 - $t = 10 \text{ ms}$

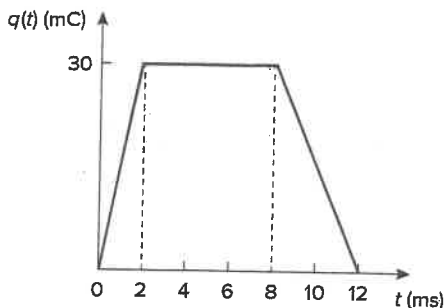


Figure 1.23
For Prob. 1.6.

- 1.7 The charge flowing in a wire is plotted in Fig. 1.24. Sketch the corresponding current.

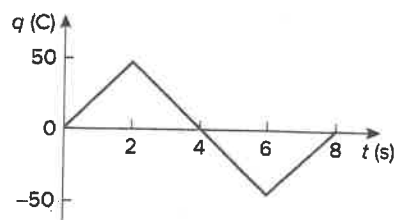


Figure 1.24
For Prob. 1.7.

- 1.8 The current flowing past a point in a device is shown in Fig. 1.25. Calculate the total charge through the point.

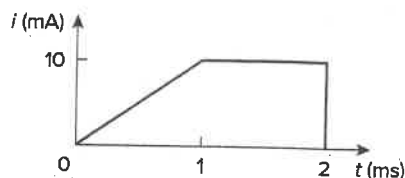


Figure 1.25
For Prob. 1.8.

- 1.9 The current through an element is shown in Fig. 1.26. Determine the total charge that passed through the element at:
- $t = 1 \text{ s}$
 - $t = 3 \text{ s}$
 - $t = 5 \text{ s}$

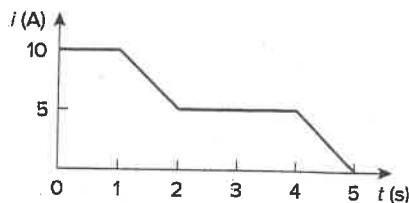


Figure 1.26
For Prob. 1.9.

Sections 1.4 and 1.5 Voltage, Power, and Energy

- 1.10 A lightning bolt with 10 kA strikes an object for $15 \text{ } \mu\text{s}$. How much charge is deposited on the object?
- 1.11 A rechargeable flashlight battery is capable of delivering 90 mA for about 12 h. How much charge can it release at that rate? If its terminal voltage is 1.5 V, how much energy can the battery deliver?
- 1.12 If the current flowing through an element is given by

$$i(t) = \begin{cases} 3t \text{ A}, & 0 \leq t < 6 \text{ s} \\ 18 \text{ A}, & 6 \leq t < 10 \text{ s} \\ -12 \text{ A}, & 10 \leq t < 15 \text{ s} \\ 0, & t \geq 15 \text{ s} \end{cases}$$

Plot the charge stored in the element over $0 < t < 20 \text{ s}$.

- 1.13 The charge entering the positive terminal of an element is

$$q = 5 \sin 4\pi t \text{ mC}$$

while the voltage across the element (plus to minus) is

$$v = 3 \cos 4\pi t \text{ V}$$

- (a) Find the power delivered to the element at $t = 0.3 \text{ s}$.
- (b) Calculate the energy delivered to the element between 0 and 0.6 s .
- 1.14 The voltage $v(t)$ across a device and the current $i(t)$ through it are

$$v(t) = 10 \cos(2t) \text{ V}, i(t) = 20(1 - e^{-0.5t}) \text{ mA}$$

Calculate:

- (a) the total charge in the device at $t = 1 \text{ s}$, $q(0) = 0$.
- (b) the power consumed by the device at $t = 1 \text{ s}$.
- 1.15 The current entering the positive terminal of a device is $i(t) = 6e^{-2t} \text{ mA}$ and the voltage across the device is $v(t) = 10di/dt \text{ V}$.
- (a) Find the charge delivered to the device between $t = 0$ and $t = 2 \text{ s}$.
- (b) Calculate the power absorbed.
- (c) Determine the energy absorbed between $t = 0$ and $t = 3 \text{ s}$.

Section 1.6 Circuit Elements

- 1.16 Figure 1.27 shows the current through and the voltage across an element.

- (a) Sketch the power delivered to the element for $t > 0$.
- (b) Find the total energy absorbed by the element for the period of $0 < t < 4 \text{ s}$.

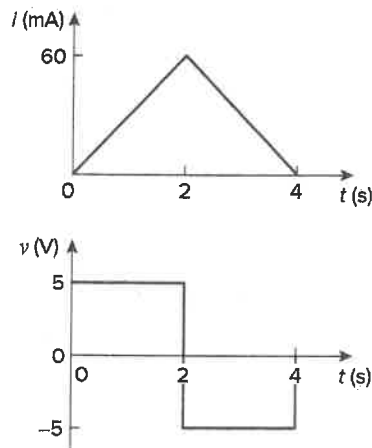


Figure 1.27
For Prob. 1.16.

- 1.17 Figure 1.28 shows a circuit with five elements. If $p_1 = -205 \text{ W}$, $p_2 = 60 \text{ W}$, $p_4 = 45 \text{ W}$, and $p_5 = 30 \text{ W}$, calculate the power p_3 absorbed by element 3.

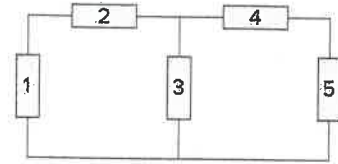


Figure 1.28
For Prob. 1.17.

- 1.18 Find the power absorbed by each of the elements in Fig. 1.29.

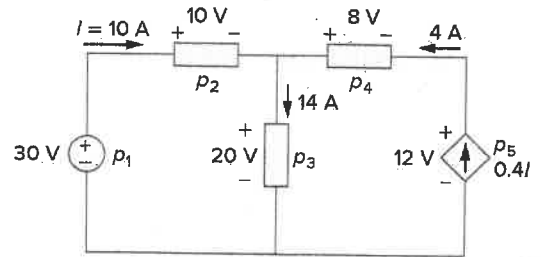


Figure 1.29
For Prob. 1.18.

- 1.19 Find i and the power absorbed by each element in the network of Fig. 1.30.

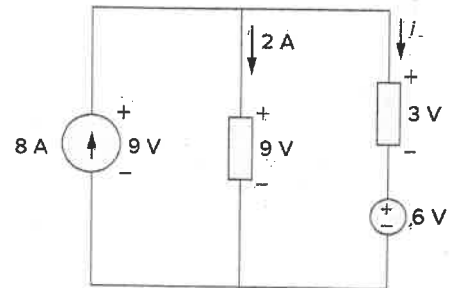


Figure 1.30
For Prob. 1.19.

- 1.20 Find V_o and the power absorbed by each element in the circuit of Fig. 1.31.

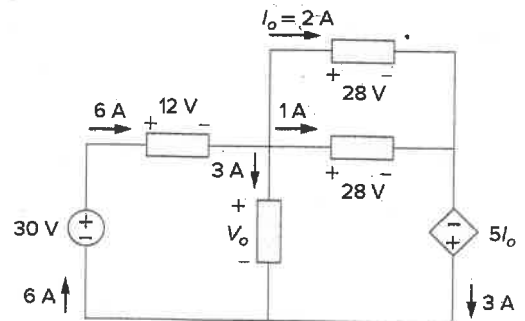


Figure 1.31
For Prob. 1.20.

Section 1.7 Applications

- 1.21 A 60-W incandescent bulb operates at 120 V. How many electrons and coulombs flow through the bulb in one day?
- 1.22 A lightning bolt strikes an airplane with 40 kA for 1.7 ms. How many coulombs of charge are deposited on the plane?
- 1.23 A 1.8-kW electric heater takes 15 min to boil a quantity of water. If this is done once a day and power costs 10 cents/kWh, what is the cost of its operation for 30 days?
- 1.24 A utility company charges 8.2 cents/kWh. If a consumer operates a 60-W light bulb continuously for one day, how much is the consumer charged?
- 1.25 A 1.5-kW toaster takes roughly 3.5 minutes to heat four slices of bread. Find the cost of operating the toaster once per day for 1 month (30 days). Assume energy costs 8.2 cents/kWh.
- 1.26 A flashlight battery has a rating of 0.8 ampere-hours (Ah) over a period of 10 hours.
- How much constant current can it deliver over 10 hours?
 - How much constant power can it deliver over the 10 hours if its terminal voltage is 6 V?
 - How much energy is stored in the battery in Wh?
- 1.27 A constant current of 3 A for 4 hours is required to charge an automotive battery. If the terminal voltage is $10 + t/2$ V, where t is in hours and we start at $t = 0$,
- how much charge is transported as a result of the charging?
 - how much energy is expended?
 - how much does the charging cost? Assume electricity costs 9 cents/kWh.
- 1.28 A 60-W incandescent lamp is connected to a 120-V source and is left burning continuously in an otherwise dark staircase. Determine:
- the current through the lamp,
 - the cost of operating the light for one non-leap year if electricity costs 9.5 cents per kWh.
- 1.29 An electric stove with four burners and an oven is used in preparing a meal as follows.
- | | |
|----------------------|----------------------|
| Burner 1: 20 minutes | Burner 2: 40 minutes |
| Burner 3: 15 minutes | Burner 4: 45 minutes |
| Oven: 30 minutes | |
- If each burner is rated at 1.2 kW and the oven at 1.8 kW, and electricity costs 12 cents per kWh, calculate the cost of electricity used in preparing the meal.
- 1.30 Reliant Energy (the electric company in Houston, Texas) charges customers as follows:
- Monthly charge \$6
 - First 250 kWh @ \$0.02/kWh
 - All additional kWh @ \$0.07/kWh
- If a customer uses 2,436 kWh in one month, how much will Reliant Energy charge?
- 1.31 In a household, a 120-W PC is run for 4 hours/day, while a 60-W bulb runs for 8 hours/day. If the utility company charges \$0.12/kWh, calculate how much the household pays per year on the PC and the bulb.

Comprehensive Problems

- 1.32 A telephone wire has a current of $20\mu\text{A}$ flowing through it. How long does it take for a charge of 15 C to pass through the wire?
- 1.33 A lightning bolt carried a current of 2 kA and lasted for 3 ms. How many coulombs of charge were contained in the lightning bolt?
- 1.34 Figure 1.32 shows the power consumption of a certain household in 1 day. Calculate:
- the total energy consumed in kWh,
 - the average power over the total 24 hour period.

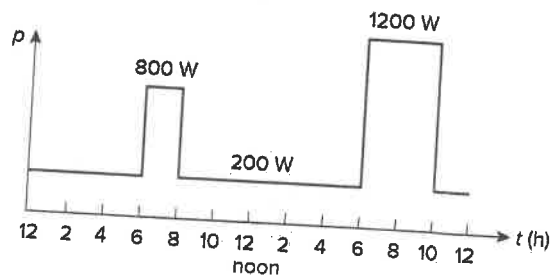


Figure 1.32
For Prob. 1.34.

- 1.35** The graph in Fig. 1.33 represents the power drawn by an industrial plant between 8:00 and 8:30 A.M. Calculate the total energy in MWh consumed by the plant.

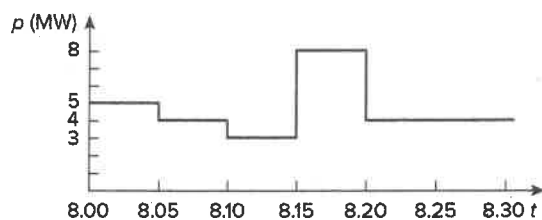


Figure 1.33
For Prob. 1.35.

- 1.36** A battery may be rated in ampere-hours (Ah). A lead-acid battery is rated at 160 Ah.
- What is the maximum current it can supply for 40 h?
 - How many days will it last if it is discharged at a rate of 1 mA?
- 1.37** A 12-V battery requires a total charge of 40 ampere-hours during recharging. How many joules are supplied to the battery?
- 1.38** How much energy does a 10-hp motor deliver in 30 minutes? Assume that 1 horsepower = 746 W.
- 1.39** A 600-W TV receiver is turned on for 4 h with nobody watching it. If electricity costs 10 cents/kWh, how much money is wasted?